Analysis and Design of Kemp-Type 3 dB Quadrature Couplers

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Abstract —An analysis is presented of the Kemp *et al.* type of 3 dB quadrature coupler. A table of element values is given for three-, five-, and seven-section couplers for various ripple levels. It is shown that the Kemp approach overcomes the tight coupling problem associated with the Cristal and Young method, but that it introduces a bandwidth contraction penalty.

I. INTRODUCTION

Use is made of 3 dB quadrature couplers in many microwave systems and components, e.g. balanced amplifiers and power combiner/dividers, and they are now required to operate over decade bandwidths, e.g. 2–20 GHz. If the bandwidth of the coupler is defined, arbitrarily, as being when the coupling remains within a 3 ± 1 dB window, then the basic single-section 3 dB coupler has a bandwidth of 2.3:1. Since the coupling is a monotonically decreasing function of frequency away from band center for a single-section coupler, greater bandwidth can be obtained by using a 2 dB coupler [1], whence the bandwidth increases to 3.8:1 but is still well short of the required 10:1. In microstrip 2 and 3 dB quadrature couplers are usually implemented in the form proposed by Lange [2].

The obvious solution to achieving broader bandwidth is to use a symmetrical multisection coupler, and Cristal and Young [3] have presented detailed design data for this type of device. However, as is well known, the required coupling of the central element(s) in multisection couplers is invariably tighter than that of the overall coupler; e.g., a three-section 3 ± 1 dB coupler requires the central element to have a coupling of 0.89 dB.

Such tight coupling values are impossible to realize in microstrip using a multifingered Lange arrangement.

In 1983 Kemp *et al.* [4] described a method of overcoming the tight coupling problem associated with multisection couplers but gave little design information. This type of coupler is used by at least one company in its commercial products. This paper provides a detailed analysis of and design data for Kemp-type couplers.

II. ANALYSIS AND DESIGN

A symmetrical multisection coupler is shown in Fig. 1(a), together with its even-mode equivalent circuit with respect to the symmetry plane AA'. The coupled and direct port responses are determined by, respectively, the reflection and transmission coefficients of the even-mode network, i.e.,

$$|S_{12}|^2 = |\gamma_e|^2 \qquad |S_{14}|^2 = |\tau_e|^2. \tag{1}$$

Since the network is lossless,

$$|\gamma_e|^2 + |\tau_e|^2 = 1. \tag{2}$$

The coupler must be symmetrical in order to ensure phase quadrature between the two outputs. Straightforward analysis

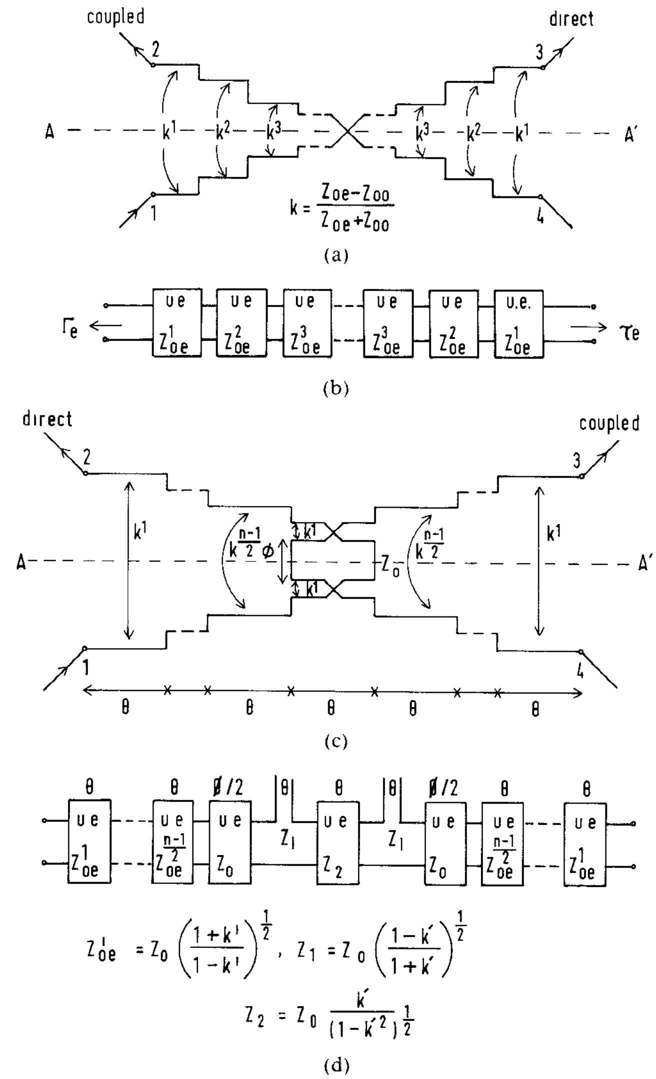

Fig. 1. Cristal and Young type of multisection quadrature coupler. (a) Physical layout. (b) Even-mode equivalent circuit. Kemp type of coupler (c) Physical layout. (d) Even-mode equivalent circuit.

shows that, for an arbitrary cascade of n unit elements, $|\tau_e|^2$ must be expressible in the form

$$|\tau_e|^2 = \frac{1}{1 + [P_n(\sin\Theta)]^2} \tag{3}$$

and the necessary and sufficient condition for (3) to be realized by a symmetrical network is that P_n be an odd polynomial of degree n in $\sin\Theta$. Unfortunately, there is no closed-form analytic solution for P_n or for the element values for an equiripple response, so Cristal and Young provided an extensive set of tables.

The basic idea behind the Kemp *et al.* type of coupler is the observation that a tandem connection of two couplers produces identical coupling (but only at band center) to that of a single coupler provided that

$$k'^2 = \frac{1 \pm \sqrt{1-k^2}}{2} \tag{4}$$

where k^2 and k'^2 are the power coupling coefficients. This

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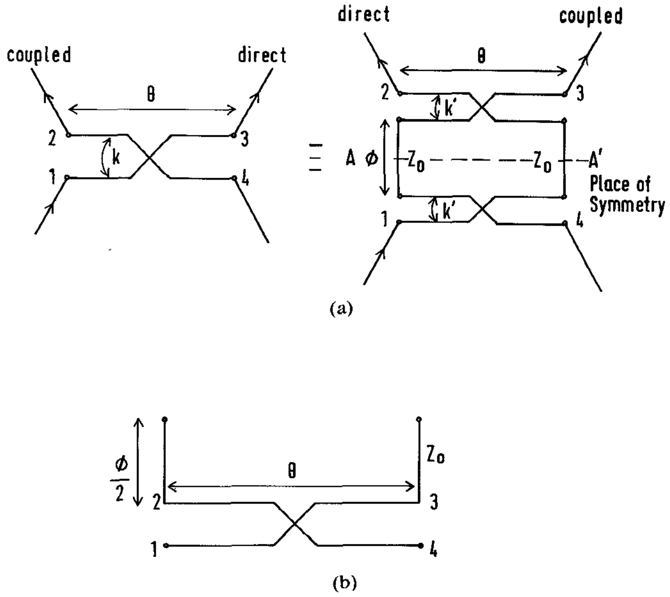

Fig. 2. (a) Equivalence between a tandem section and a single-section coupler. (b) Even-mode equivalent circuit of tandem section.

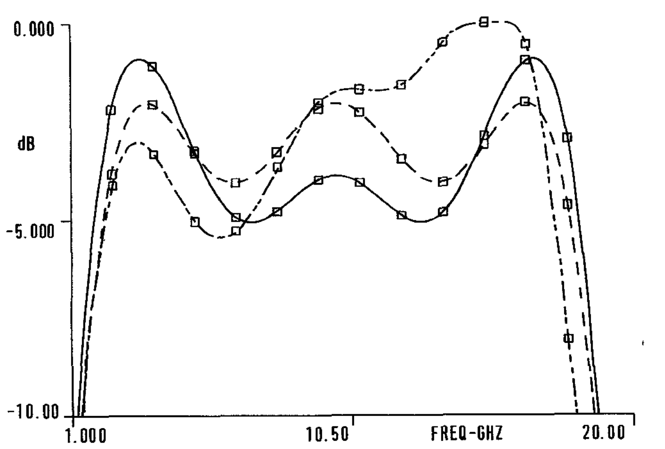

Fig. 3. Computed results for five-section coupler: ---- ideal ($\phi = 0$); ——— distortion resulting from $\phi = 45°$; ______ effect of direct insertion of Fig. 2 equivalence into Cristal and Young coupler.

equivalence is illustrated in Fig. 2(a). In this paper a slight generalization has been made whereby an arbitrary length of transmission line of characteristic impedance Z_0 is inserted between the two couplers so that unwanted direct coupling between the two couplers can be eliminated. This has no effect upon the amplitude response of the tandem coupler or upon the phase quadrature property,

Equation (4) has two solutions, one a narrow-band equivalence and the other broad-band. It is indeed fortunate that the broadband equivalence occurs when the minus sign is chosen since this results in $k'^2 < k^2$; i.e., the fabrication difficulties are eased. Note that the direct and coupled ports are interchanged.

An n-section Kemp *et al.* type of coupler is obtained by replacing the extremely tightly coupled central section of Fig. 1(a) with its tandem equivalent as shown in Fig. 1(c). While in principle it might be thought necessary to use more than one tandem section, the numerical results to follow show that one is sufficient for all the cases considered. Again, note the interchange of the direct and coupled ports between Fig. 1(a) and (c). Since the "equivalence" of Fig. 2(a) is not exact with respect to either frequency or ports, its direct insertion into Fig. 1(a) leads to a considerable distortion of the overall coupling response. Fig. 3 illustrates the distortion which occurs for a five-section 3 ± 1 dB coupler.

Fig. 1(d) shows the even-mode equivalent circuit of the Kemp type coupler with respect to the symmetry plane AA'. The derivation of the even-mode equivalent circuit for the central tandem section is given in the Appendix. The coupled port response is determined by the *transmission* coefficient of the even-mode network of Fig. 1(d) while the coupled port response of Cristal and Young's coupler is determined by the *reflection* coefficient of Fig. 1(b). Thus, the element values of the Kemp type of coupler are completely unrelated to those of Cristal and Young. Assuming for the moment that $\phi = 0$ then application of Kuroda's transformations [5] to Fig. 1(d) shows that one of the stubs is electrically redundant and hence the coupled port response must be expressible in the form

$$|S_{13}|^2 = \frac{1}{1+\left(\dfrac{P_{1+n}(\sin\Theta)}{\sin\Theta}\right)^2}. \tag{5}$$

The necessary and sufficient condition for this expression to be realizable by a symmetrical network is that $P_{1+n}(\sin\Theta)$ be an even polynomial in $\sin\Theta$. There is no closed-form analytic solution for $P_{1+n}(\sin\Theta)$ to achieve an equiripple response, but the similarity of (3) and (5) shows that the numerical procedure used by Cristal and Young can be used in this case also with suitable modifications.

Table I lists the computed coupling values required for three-, five-, and seven-section 3 dB couplers for various ripple levels. The corresponding values from Cristal and Young are also given for comparison. It can be seen that the tight coupling problem is completely eliminated. It can be seen from Table I that there is a price to be paid for overcoming the tight coupling problem, namely that there is a considerable reduction in bandwidth; in fact, it is more than halved in some instances. The cause of the bandwidth reduction is the presence of the series open-circuit stubs in the even-mode equivalent circuit. A three-section coupler provides a useful increase in bandwidth over the single-section 2 or 3 dB couplers mentioned in Section I, but the maximum number of sections is probably limited to 7 since the coupling values of the couplers in the tandem section for a seven-section device are such that they would need to be realized in a Lange form, which would make the practical realization somewhat cumbersome.

Thus far it has been assumed that $\phi = 0$ but in practice one would like $\phi \neq 0$ to eliminate unwanted cross-coupling. If a microstrip realization is required, then the couplers need to be at least three substrate thicknesses apart in order to have negligible cross-coupling. Suppose a 0.015-in-thick alumina substrate is used and the center frequency is 10 GHz; then $\phi = 45°$ at 10 GHz would satisfy the above criterion. Unfortunately, this significantly degrades the coupling response, as Fig. 3 shows. Hence it is concluded that much smaller values for ϕ must be used, which implies that cross-coupling will be a problem or that computer optimization of the lengths and couplings is required.

An academically interesting situation arises if $\phi = \Theta$, since this is the only example of which the author is aware of a lossless distributed network with all transmission lines having the same length that does not have a periodic response about $\Theta = 90°$. Analysis shows that this network is periodic about $\Theta = 180°$ and

TABLE I
PARAMETERS FOR 3 dB KEMP *et al.* TYPE COUPLERS

3 section

Ripple, dB	C_1=C_3, dB	C_2, dB	W%	B
0.1	19.97 (16.09)	7.0 (1.64)	86 (101)	2.51 (3.03)
0.25	17.63 (14.0)	6.73 (1.44)	104 (123)	3.17 (4.18)
0.5	15.24 (11.82)	6.42 (1.21)	120 (141)	4.0 (5.81)
1.0	12.28 (8.95)	5.95 (0.89)	136 (161)	5.25 (9.20)

5 Section

Ripple, dB	C_1=C_5, dB	C_2=C_4, dB	C_3, dB	W%	B
0.1	23.1 (22.45)	13.67 (10.27)	6.24 (1.10)	116 (133)	3.76 (4.93)
0.25	19.6 (18.74)	12.13 (8.72)	5.98 (0.94)	130 (149)	4.71 (6.89)
0.5	16.3 (15.35)	10.65 (7.24)	5.70 (0.77)	142 (162)	5.90 (9.63)
1.0	12.5 (11.35)	8.88 (5.38)	5.30 (0.55)	153 (175)	7.51 (15.24)

7 Section

Ripple, dB	C_1=C_7, dB	C_2=C_6, dB	C_3=C_5, dB	C_4, dB	W%	B
0.1	25.2 (25.84)	15.8 (15.53)	10.55 (7.50)	5.71 (0.82)	134 (150)	5.06 (6.95)
0.25	20.85 (21.13)	13.53 (12.94)	9.42 (6.31)	5.47 (0.69)	146 (163)	6.41 (9.73)
0.5	17.16 (17.07)	11.48 (10.62)	8.36 (5.21)	5.22 (0.56)	154 (173)	7.70 (13.56)
1.0	12.9 (12.49)	9.1 (7.85)	7.1 (3.83)	4.85 (0.39)	162 (182)	9.53 (21.41)

$C_i = 10 \log_{10} k_i^2$

the reason is immediately evident from the even-mode network in Fig. 1(d).

An alternative method of achieving ultra-broad-band 3 dB quadrature couplers is to use either a discrete or a continuous tandem multisection construction [6], [7]. This technique would seem to have two distinct advantages over the Kemp approach; namely, the bandwidth contraction problem is eliminated, allowing decade bandwidths to be achieved, and the two couplers can be separated by an arbitrary amount to reduce cross-coupling without affecting the amplitude response because the interconnection occurs at the exterior rather than the interior ports (see the Appendix).

III. CONCLUSIONS

An analysis has been presented of the Kemp *et al.* type of 3 dB quadrature coupler, and a table of element values has been given. It has been shown that the Kemp approach overcomes the tight coupling problem associated with the Cristal and Young method but there is a severe bandwidth contraction penalty. It has also been shown that the use of interconnection lines sufficiently long to reduce cross-coupling causes an unacceptable degradation in coupling performance. Finally, although the technique provides a useful increase in bandwidth over that obtainable from a single-section coupler, it is suggested that the tandem multisection method is more suitable for decade bandwidths.

APPENDIX
DERIVATION OF EVEN-MODE EQUIVALENT CIRCUIT OF THE CENTRAL TANDEM SECTION

With reference to Fig. 2(b) one has

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{24} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \tag{A1}$$

with

$$a_2 = e^{-j\phi} b_2 \qquad a_3 = e^{-j\phi} b_3. \tag{A2}$$

Also, for the case considered here,

$$S_{24} = S_{13}, S_{34} = S_{12}. \tag{A3}$$

Substituting (A2) and (A3) into (A1) results in

$$\begin{pmatrix} b_1 \\ b_4 \end{pmatrix} = e^{-j\phi} \begin{pmatrix} S_{12}^2 + S_{13}^2 & 2S_{12}S_{13} \\ 2S_{12}S_{13} & S_{12}^2 + S_{13}^2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_4 \end{pmatrix}. \tag{A4}$$

The equivalent circuit of Fig. 2(b) with $\phi = 0$ is known (see, for example, [5]) and (A4) shows that the effect of a nonzero value for ϕ is to introduce a unit element of length $\phi/2$ and characteristic impedance Z_0 at the input and output of the known circuit.

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Percutaneous Transluminal Microwave Balloon Angioplasty

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***Abstract*—Microwave balloon angioplasty (MBA) combines conventional balloon angioplasty techniques with microwave heating to help enlarge the lumen of narrowed arteries and reduce the occurrence of**

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